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# B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2024

## Second Semester

Complementary Course—INTEGRAL CALCULUS AND MATRICES

(Common for B.Sc. Physics, Chemistry, Petrochemicals, Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

[2013—2016 Admissions]

Time : Three Hours

Maximum Marks : 80

## Part A

*Answer* **all** *questions.*
*Each question carries* 1 *mark.*

1. Given that $\int_1^4 f(x)dx = -2$, then what is $\int_4^1 f(x)dx$.

2. State mean value theorem for definite integrals.

3. State fundamental theorem for integral calculus.

4. Given that $f$ is a continuous function on an internal $[-a,a]$ and $f$ is an odd function, then what is

   $\int_{-a}^{a} f(x)dx$ ?

5. Write the formula for finding the length of a continuously differentiate curve $y = f(x), a \le x \le b$.

6. Write the formula for finding the surface area for revolution about $x$-axis of a continuously differentiate function $f(x) \ge 0$ on $[a,b]$.

7. Evaluate $\int_1^2 \int_y^{y^2} dx\, dy$.



Turn over

8. Write the formula for finding the area of a closed bounded region R in polar coordinates.

9. What is a non singular matrix ?

10. State Cayley-Hamilton theorem.

(10 × 1 = 10)

## Part B

*Answer any* **eight** *questions.*
*Each question carries 2 marks.*

11. Suppose that $\int_{-3}^{0} g(t)dt = \sqrt{2}$. Find $\int_{0}^{-3} g(t)dt$ and $\int_{-3}^{0} [-g(x)]dx$.

12. Evaluate $\int_{0}^{2} \left(3x^2 + x - 5\right)dx$ .

13. Show that if $f$ is continuous on $[a,b], a \neq b$ and if $\int_{a}^{b} f(x)dx = 0$, then $f(x) = 0$ at least once in $[a,b]$.

14. Evaluate $\int_{\pi/4}^{\pi/2} \cot\theta \operatorname{cosec}^2\theta \, d\theta$.

15. The circle $x^2 + y^2 = a^2$ is rotated about the $x$-axis to generate a sphere. Find its volume.

16. Find the length of the circle of radius $r$ defined parametrically by $x = r\cos t$, and $y = r\sin t$, $0 \leq t \leq 2\pi$.

17. Integrate $f(x,y) = x/y$ over the region in the first quadrant bounded by the lines $y = x, y = 2x, x = 1, x = 2$.

18. Find the average value of $f(x,y) = \sin(x+y)$ over the rectangle $0 \leq x \leq \pi, 0 \leq y \leq \pi$.

19. Evaluate $\int_0^1\int_0^1\int_0^1 (x^2+y^2+z^2)\,dz\,dy\,dx$.

20. Write the elementary transformations of a matrix.

21. Define row equivalent Canonical matrix.

22. Obtain the row equivalent Canonical matrix of $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{bmatrix}$.

(8 × 2 = 16)

## Part C

*Answer any* **six** *questions.*
*Each question carries* 4 *marks.*

23. Graph the integral and use area to evaluate $\int_{1/2}^{3/2} (-2x+4)\,dx$.

24. Use fundamental theorem to find $\frac{dy}{dx}$ if $y = \int_x^5 3t\sin t\,dt$.

25. Find the area of the region enclosed by the parabola $y = 2-x^2$ and the line $y = -x$.

26. Find the length of the curve :

$$y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, 0 \le x \le 1.$$

27. Find the area of the surface generated by revolving $y = x^3, 0 \le x \le \frac{1}{2}$ about the $x$-axis.

28. Sketch the region of integration and evaluate the integral $\int_1^3\int_0^2 (4-y^2)\,dy\,dx$.

29. Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

30. Find the rank of the matrix :

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 0 & 1 \\ 4 & 1 & 4 & 7 \end{bmatrix}.$$

31. Prove that the rank of the transpose of a matrix is the same as that of the original matrix.

(6 × 4 = 24)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 15 *marks.*

32. (a) Find the total area between the region $y = 3x^2 - 3, -2 \le x \le 2$ and the $x$-axis.

(b) Evaluate $\int_{\pi/2}^{0} \frac{1+\cos 2t}{2}\, dt$.

33. (a) Find the volume of the solid generated by revolving the regions bounded by $y = x^3, y = 0, x = 2$.

(b) Find the area of the surface generated by revolving $x = \cos t, y = 2 + \sin t, 0 \le t \le 2\pi$ about the $x$-axis.

34. (a) Evaluate $\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy$.

(b) Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

35. (a) Find the eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(b) Solve the system of equations :

$$\begin{aligned} 3x_1 - 7x_2 + 4x_3 &= 10 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ x_1 + x_2 + 2x_3 &= 8 \end{aligned}$$

(2 × 15 = 30 marks)